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Continuous-time Graph Representation with Sequential Survival Process

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Introduction

Graph representation learning (GRL) aims to encode the structure of a given network into low-dimensional vectors.
Applications: classification, community detection and link prediction.

The existing GRL methods have mainly addressed static networks. However, many real networks evolve through time with newly arriving nodes or emerging connections.

In recent years, there have been some efforts to model dynamic complex networks, but most approaches have focused on either
i) Discrete-time temporal networks or
ii) Continuous-time networks consisting of instantaneous link event times.

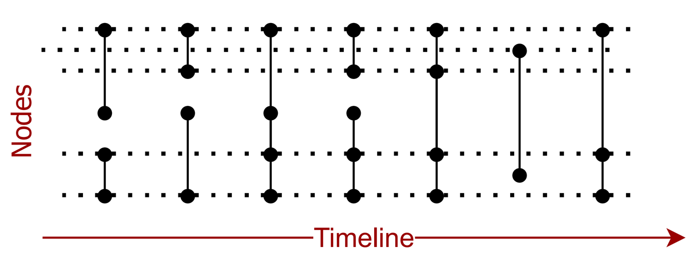


Illustration of discrete-time temporal networks

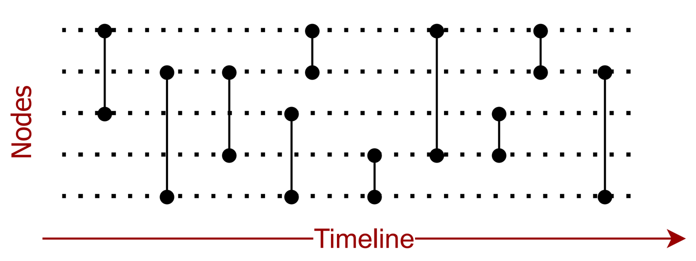
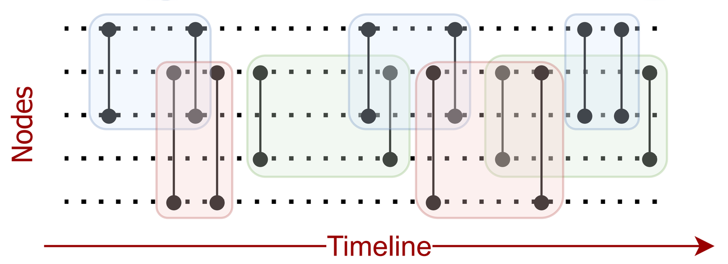


Illustration of continuous-time temporal networks

Many continuous-time dynamic real networks have intermittent linkage structures. The links consecutively emerge and disappear over time.

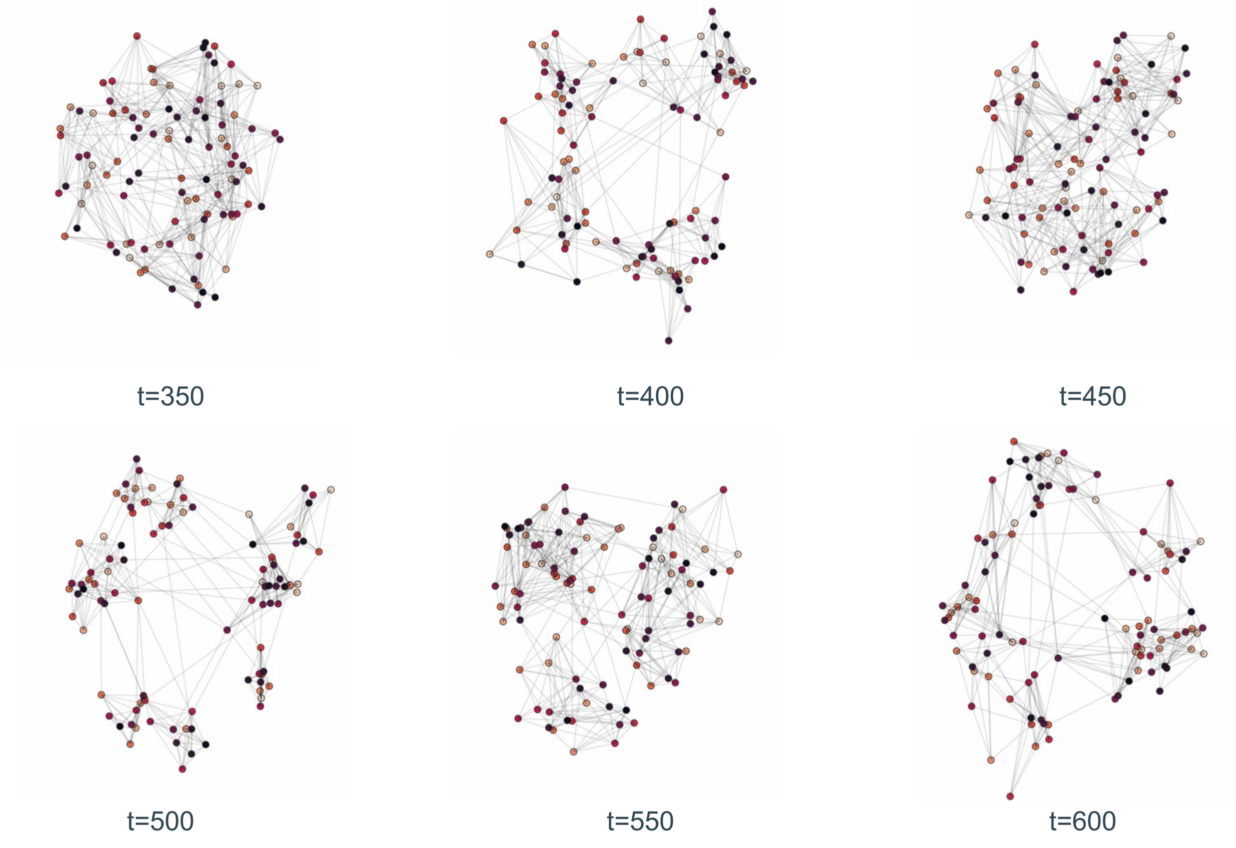
- They only account for the event time of a link but not its persistence.
- Static models also assume constant and steady relationships.



Continuous-time Intermittent Edge Persistent Graphs.

In this work

- We propose a novel counting process called ***Sequential Survival Process***.
- We learn latent node representations of **continuous-time intermittent edge persistent graphs.**
- Our approach outperforms the baselines on a wide range of datasets.
- Our approach enables continuous-time graph visualization using low-dimensional (D=2) dynamic latent representations.



Snapshots of the continuous-time embeddings for various timestamps over Synthetic-β

Proposed Approach

Survival Analysis

$$S(t) := \mathbb{P}\{T > t\} = \int_t^{\infty} f(u) = 1 - F(t) \qquad S(t) = \exp\left(-\int_0^t \lambda(t')dt'\right).$$

Sequential Survival Process

$$p_M(m) = \int_{\boldsymbol{\xi}\in\mathcal{R}} \prod_{n=1}^{m} \frac{\int_{\xi_{n-1}}^{\xi_n} \lambda(s_n, t')\mathrm{d}t'}{\exp\left(\int_{\xi_{n-1}}^{\xi_n} \lambda(s_n, t')\mathrm{d}t'\right)} \mathrm{d}\boldsymbol{\xi}$$

Objective function

$$\mathcal{L}(\Omega|\mathcal{G}) := \log p(\mathcal{G}|\Omega) = \sum_{i,j\in\mathcal{V}} \sum_{m=1}^{|\mathcal{E}_{ij}|} \left(\log \lambda_{ij}(s_m, e_m) - \int_{e_m}^{e_{m+1}} \lambda_{ij}(s_m, t)\mathrm{d}t\right)$$

Hazard function

$$\lambda_{ij}(s,t) := \exp\left(\beta(s) + s\|\mathbf{r}_i(t) - \mathbf{r}_j(t)\|^2\right).$$

s : State of pair
$\beta(s)$: Global bias term
$\mathbf{r}_i(t)$: Embedding of node i at time t

Lemma 2.1. *Let $e_0 = 0 < e_1 < \cdots < e_{M-1} < T$ be a sequence following a Sequential Survival process for node pair $(i,j) \in \mathcal{V}^2$. Then, the average squared distance between nodes during interval $[e_m, e_{m+1})$ associated with survival function $S_m(\cdot)$ and state $s_m \in \{-1, 1\}$ can be bounded by*

$$b(-1) \leq \frac{1}{(e_{m+1} - e_m)} \int_{e_m}^{e_{m+1}} \|\mathbf{r}_i(t) - \mathbf{r}_j(t)\|^2 dt \leq b(+1)$$

where $b(s) := -2s\log(e_{m+1} - e_m) + \log S(e_{m+1}) - s\beta(s)$.

We define the representation of node $i \in \mathcal{V}$ at time t based on a linear model:

$$\mathbf{r}_i(t) := \mathbf{x}_i + \mathbf{v}_i t$$

Piecewise Approximation

$$\mathbf{r}_i(t) := \mathbf{x}_i^{(0)} + \Delta_B \mathbf{v}_i^{(1)} + \Delta_B \mathbf{v}_i^{(2)} + \cdots + \Delta_B \mathbf{v}_i^{(b)} + \cdots + \mathrm{mod}(t, \Delta_B)\mathbf{v}_i^{(\lfloor t/\Delta_B \rfloor + 1)}$$

Prior distribution

$$\mathrm{vect}(\mathbf{v}) \sim \mathcal{N}(\mathbf{0}, \lambda^2\Sigma) \text{ and } \Sigma := \mathrm{diag}(\boldsymbol{\sigma}_B \otimes \boldsymbol{\sigma}_N \otimes \boldsymbol{\sigma}_D)$$

Experiments

Datasets

	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}_{min}\rvert$	$\lvert\mathcal{E}_{max}\rvert$	$\lvert\mathcal{E}\rvert$	Resolution
Synthetic-α	100	1	18	4286	N/A
Synthetic-β	100	2	12	8300	N/A
Contacts	92	1	197	5313	Second
HyperText	113	1	133	10450	Second
Infectious	410	1	29	9827	Second
Facebook	461	1	1	10222	Second
NeurIPS	327	1	6	1940	Year

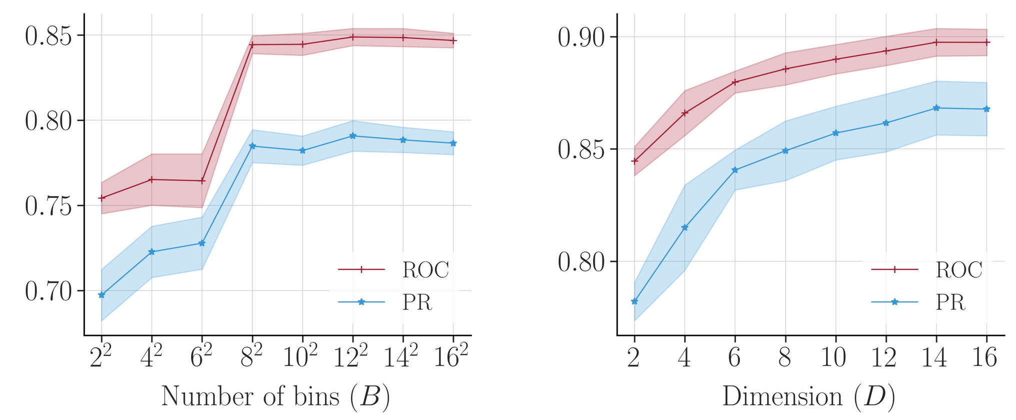
Network Completion

		LDM	Node2Vec	CTDNE	HTNE	M^{2}DNE	PiVeM	GraS^{2}P
Synthetic-α	ROC	.711 ± .004	.743 ± .002	.692 ± .007	.698 ± .021	.558 ± .008	.744 ± .002	**.810 ± .009**
	PR	.630 ± .006	.667 ± .009	.650 ± .007	.645 ± .019	.582 ± .004	.653 ± .004	**.751 ± .011**
Synthetic-β	ROC	.491 ± .020	.534 ± .008	.502 ± .008	.525 ± .004	.517 ± .013	.593 ± .006	**.677 ± .018**
	PR	.486 ± .016	.498 ± .007	.502 ± .010	.517 ± .006	.522 ± .015	.587 ± .011	**.646 ± .022**
Contacts	ROC	.508 ± .008	.584 ± .004	.564 ± .034	.472 ± .024	.486 ± .013	.493 ± .006	**.680 ± .013**
	PR	.490 ± .004	.555 ± .023	.543 ± .036	.477 ± .023	.500 ± .008	.492 ± .016	**.641 ± .023**
HyperText	ROC	.541 ± .015	.533 ± .012	.462 ± .016	.441 ± .017	.461 ± .021	.426 ± .013	**.692 ± .010**
	PR	.503 ± .010	.490 ± .013	.477 ± .016	.449 ± .009	.479 ± .023	.437 ± .007	**.656 ± .024**
Infectious	ROC	.689 ± .007	.671 ± .003	.639 ± .006	.653 ± .013	.554 ± .005	.669 ± .004	**.742 ± .026**
	PR	.615 ± .007	.601 ± .005	.593 ± .005	.596 ± .010	.560 ± .009	.598 ± .004	**.673 ± .024**
Facebook	ROC	.717 ± .004	.675 ± .001	.539 ± .005	.608 ± .001	.570 ± .010	.710 ± .002	**.723 ± .010**
	PR	.659 ± .006	.603 ± .005	.538 ± .013	.575 ± .001	.562 ± .009	.662 ± .002	**.671 ± .012**
NeurIPS	ROC	.679 ± .010	.697 ± .005	.558 ± .020	.654 ± .025	.531 ± .005	**.748 ± .010**	.735 ± .029
	PR	.618 ± .016	.606 ± .020	.552 ± .025	.613 ± .026	.553 ± .011	**.761 ± .020**	.749 ± .021

Future Link Prediction

		LDM	Node2Vec	CTDNE	HTNE	M^{2}DNE	PiVeM	GraS^{2}P
Synthetic-α	ROC	.748 ± .007	.756 ± .005	.652 ± .012	.784 ± .013	.654 ± .011	.740 ± .007	**.902 ± .011**
	PR	.719 ± .012	.700 ± .020	.636 ± .019	.800 ± .016	.745 ± .008	.741 ± .005	**.918 ± .008**
Synthetic-β	ROC	.515 ± .018	.538 ± .004	.503 ± .020	.560 ± .006	.519 ± .012	**.894 ± .005**	.880 ± .012
	PR	.525 ± .021	.501 ± .007	.494 ± .016	.548 ± .004	.554 ± .014	**.845 ± .007**	.843 ± .014
Contacts	ROC	**.821 ± .004**	.703 ± .002	.635 ± .013	.727 ± .002	.590 ± .002	.692 ± .005	.793 ± .013
	PR	**.773 ± .005**	.648 ± .006	.599 ± .014	.689 ± .004	.610 ± .006	.675 ± .004	.752 ± .019
HyperText	ROC	**.663 ± .004**	.553 ± .003	.503 ± .010	.530 ± .018	.548 ± .004	.559 ± .003	.654 ± .005
	PR	.609 ± .003	.516 ± .008	.503 ± .006	.518 ± .014	.529 ± .008	.534 ± .002	**.612 ± .010**
Infectious	ROC	**.958 ± .004**	.869 ± .002	.847 ± .008	.893 ± .013	.655 ± .008	.945 ± .006	.943 ± .017
	PR	**.943 ± .008**	.818 ± .007	.820 ± .014	.853 ± .007	.698 ± .009	.932 ± .006	.923 ± .025
Facebook	ROC	**.781 ± .007**	.694 ± .003	.564 ± .005	.626 ± .003	.609 ± .015	.775 ± .002	.705 ± .009
	PR	.765 ± .009	.653 ± .004	.557 ± .004	.599 ± .011	.603 ± .011	**.766 ± .003**	.648 ± .009
NeurIPS	ROC	.682 ± .019	.695 ± .012	.637 ± .007	.676 ± .014	.661 ± .006	.623 ± .010	**.820 ± .008**
	PR	.634 ± .024	.621 ± .018	.615 ± .015	.635 ± .025	.674 ± .014	.628 ± .006	**.788 ± .018**

Influence of the hyper-parameters



Influence of the hyper-parameters for the network reconstruction task over Synthetic-α

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